Figure 5.5 shows the components of an actual refrigeration cycle.

Another difference between an actual cycle and the Carnot cycle is the composition of the refrigerant at state 1. In the Carnot cycle, the refrigerant at state 1 is a two-phase mixture; in an actual cycle, it is a vapor. The process of compressing a liquid-vapor mixture is called *wet compression*. Wet compression is avoided because it increases both the amount of wear on the compressor and the risk of damaging it. The process of compressing pure vapor is called *dry compression*.

### 6. VAPOR COMPRESSION REFRIGERATION CYCLE

Figure 5.6 shows the *T*-s diagram for an ideal steady-state vapor compression cycle. (This is considered the ideal form of this cycle even though the throttling process from state 3 to state 4 contains irreversibilities.) For the analysis of a vapor compression refrigeration system, kinetic and potential energy changes are neglected.

The processes of the ideal vapor compression refrigeration cycle are as follows.

- **process 1-2s**: isentropic compression of refrigerant in gas state
- **process 2s-3**: heat transfer from condenser to surroundings (both heat removed from cold space and heat gained from compression); refrigerant leaves condenser in liquid state
- **process 3-4s**: isentropic throttling process to two-phase liquid-vapor mixture in state 4s; temperature of refrigerant is reduced, allowing heat to be removed from cold space
- **process 4s-1**: heat transfer from cold space to refrigerant in evaporator

### 7. IDEAL VAPOR COMPRESSION CYCLE PERFORMANCE

A theoretical limit for a vapor compression refrigeration cycle can be obtained by regarding as negligible both the irreversibilities of the system components (compressor, evaporator, and condenser) and the frictional losses encountered as the refrigerant flows through the system piping. The heat transfer from the compressor to the surroundings is ignored, making it an isentropic process.

The evaporator’s function is to remove heat from the cold space. As heat is removed from the cold space, the refrigerant flowing through the evaporator is vaporized. Applying the first law of thermodynamics to the evaporator gives

\[
\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4)
\]

The heat transfer rate, \(\dot{Q}_{\text{in}}\), is the refrigeration capacity of the cycle. A common unit of measure of refrigeration capacity is the ton of refrigeration, which is equivalent to 200 Btu/min (211 kW).

After evaporation, the refrigerant goes to the compressor as a saturated or superheated vapor. Assuming that kinetic and potential energy changes are negligible and that compression is adiabatic, the first law applied to the compressor gives

\[
\dot{W}_{\text{compressor}} = \dot{m}(h_2 - h_1)
\]

After compression, the refrigerant flows to the condenser, where the heat adsorbed by the refrigerant and the heat of compression are rejected. With the same assumptions, the first law applied to the condenser gives

\[
\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3)
\]

After leaving the condenser, the refrigerant is expanded through the expansion valve. The expansion process is analyzed as a throttling process. A throttling process is
considered a constant enthalpy process, so the first law gives
\[ h_4 = h_3 \]

The expansion process is irreversible, so the entropy of the refrigerant increases during the process. After expansion, the refrigerant is a two-phase mixture.

The performance of the cycle is defined as the coefficient of performance, the ratio of the heat removed from the colder space to the work put into the system.

\[ \beta_{\text{ideal}} = \frac{\dot{Q}_{\text{in}}}{W_{\text{compressor}}} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{h_1 - h_4}{h_2 - h_1} \]  

8. ACTUAL VAPOR COMPRESSION CYCLE PERFORMANCE

The actual refrigeration cycle is shown in Fig. 5.7. The line 1-2 accounts for the reversibilities of the compression process, but does not consider the possible heat transfer to the surroundings. Compared to the ideal cycle in Fig. 5.6, the actual cycle requires more work input (compression) to achieve the equivalent refrigeration effect. Based on Eq. 5.7, the actual cycle will have a lower coefficient of performance than the ideal cycle because of the greater amount of work needed.

\[ \beta = \frac{\dot{Q}_{\text{in}}}{W_{\text{compressor A}} + W_{\text{compressor B}}} \]

9. CASCADE REFRIGERATION SYSTEMS

A cascade refrigeration system joins two or more refrigeration cycles in series. Such a system can make possible a large difference between the temperatures of the warm and cold spaces (\( T_H \) and \( T_C \)).

Figure 5.8 shows a cascade refrigeration system. It consists of two refrigeration cycles, cycle A and cycle B, that come into contact through a counterflow heat exchanger. The desired refrigeration effect is produced by the evaporation step of cycle A. In cycle B’s evaporation step, cycle B absorbs the heat rejected from cycle A. The condenser step in cycle B rejects the heat of both cycles to the surroundings.

10. GAS REFRIGERATION SYSTEMS

In a gas refrigeration system, the refrigerant does not change phase but remains in the gas phase throughout
From Fourier’s law (see Eq. 8.8), the heat transfer in the
\(x\)-direction can be written as
\[
q_x = -k A_c \frac{dT}{dx} \quad 8.131
\]
The cross-sectional area, \(A_c\), can vary with \(x\). The heat
conduction at the location \(x + dx\) is
\[
q_{x+dx} = q_x + \frac{dq_x}{dx} \, dx \quad 8.132
\]
Substituting Eq. 8.131 twice into Eq. 8.132,
\[
q_{x+dx} = -k A_c \frac{dT}{dx} - k \frac{d}{dx} \left( A_c \frac{dT}{dx} \right) \, dx \quad 8.133
\]
The convective heat transfer rate can be written as
\[
dq_{\text{conv}} = h \, dA_c(T - T_\infty) \quad 8.134
\]
The differential surface area of the element under con-
consideration is \(dA_c\). Equation 8.131, Eq. 8.133, and
Eq. 8.134 can be substituted into Eq. 8.130 to give
\[
-k A_c \frac{dT}{dx} = -k A_c \frac{dT}{dx} - k \frac{d}{dx} \left( A_c \frac{dT}{dx} \right) \, dx \\
+ h \, dA_c(T - T_\infty) \quad 8.135
\]
Simplifying,
\[
k \frac{d}{dx} \left( A_c \frac{dT}{dx} \right) \, dx - h \, dA_c(T - T_\infty) = 0
\]
\[
\frac{dA_c}{dx} \frac{dT}{dx} + A_c \frac{d^2T}{dx^2} - h \frac{dA_c}{dx}(T - T_\infty) = 0
\]
\[
d^2T + \frac{1}{A_c} \frac{dT}{dx} - \left( \frac{1}{A_c} \right) \left( \frac{h \, dA_c}{dx} \right) (T - T_\infty) = 0
\]
\[
8.136
\]
Equation 8.136 represents the general form for one-
dimensional heat transfer for an extended surface. Once
boundary conditions are known, this equation can be
developed further to provide an expression for tempera-
ture distribution as a function of \(x\). The particular
geometry of the fin will also affect the development
and solution of Eq. 8.136.

16. FINS WITH UNIFORM CROSS SECTIONS

Figure 8.18 shows two examples of straight fins with
uniform cross-sectional areas. Each fin is attached to a
base with surface temperature \(T_s\), and each fin extends
into a fluid.
12 Heat Transfer Problems

Problems

1. A double-paned window is 4 ft wide and 6 ft tall. Each pane is $\frac{1}{8}$ in thick, and the two panes are separated by an air gap $\frac{1}{4}$ in thick. The film coefficient for the outer pane is 22 Btu/hr-ft$^2$-°F, and that for the inner pane is 1.5 Btu/hr-ft$^2$-°F. The window is exposed to inside air at 68°F and outside air at 15°F. The total heat transfer through the window is most nearly

(A) 24 Btu/hr
(B) 58 Btu/hr
(C) 570 Btu/hr
(D) 850 Btu/hr

2. A planar wall is composed of two materials. Wall 1 has a uniform heat generation of $1.5 \times 10^6$ W/m$^3$ and a thermal conductivity of 60 W/m-K. Wall 2 has no heat generation and a thermal resistance of 150 W/m-K. The inner surface of wall 1 is well insulated, while the outer surface of wall 2 is exposed to 30°C fluid. The temperature of the wall surface exposed to the fluid is most nearly

(A) 30°C
(B) 50°C
(C) 80°C
(D) 100°C

3. A heated titanium drum operates such that the outer surface is at a temperature of 900K and is exposed to an ambient temperature of 300K. The convection heat transfer coefficient is 80 W/m$^2$-K. Seventeen annular fins of rectangular profile are to be added to the drum as shown. The fin efficiency is 95%.

4. A sphere with a 5 mm radius is initially in equilibrium at 400°C in a furnace. The sphere is placed in 20°C air. The convection coefficient is 20 W/m$^2$-K. The thermophysical properties of the sphere’s material are

$k = 25$ W/m-K  
$c = 1000$ J/kg-K  
$\rho = 3500$ kg/m$^3$

The time it will take for the sphere to cool to 325°C is most nearly

(A) 36 s
(B) 64 s
(C) 130 s
(D) 360 s