5. For the shaft shown, the shear stress is not to exceed 110 MPa.

What is most nearly the largest torque that can be applied?

(A) 1700 N-m
(B) 1900 N-m
(C) 2300 N-m
(D) 3400 N-m

6. An aluminum (shear modulus = $2.8 \times 10^{10}$ Pa) rod is 25 mm in diameter and 50 cm long. One end is rigidly fixed to a support. Most nearly, what torque must be applied at the free end to twist the rod 4.5° about its longitudinal axis?

(A) 26 N-m
(B) 84 N-m
(C) 110 N-m
(D) 170 N-m

7. A circular bar at 10°C is constrained by rigid concrete walls at both ends. The bar is 1000 mm long and has a cross-sectional area of 2600 mm².

\[ E = \text{modulus of elasticity} \]
\[ = 200 \text{ GPa} \]
\[ \alpha = \text{coefficient of thermal expansion} \]
\[ = 9.4 \times 10^{-6} \text{ 1/°C} \]

What is the approximate axial force in the rod?

(A) 2.8 kN
(B) 15 kN
(C) 19 kN
(D) 58 kN

8. A 3 m diameter solid bar experiences opposing torques of 280 N-m at each end.

What is most nearly the maximum shear stress in the bar?

(A) 2.2 Pa
(B) 31 Pa
(C) 42 Pa
(D) 53 Pa

9. A 12.5 mm diameter steel rod is pinned between two rigid walls. The rod is initially unstressed. The rod’s temperature subsequently increases 50°C. The rod is adequately stiffened and supported such that buckling does not occur. The coefficient of linear thermal expansion for steel is $11.7 \times 10^{-6} \text{ 1/°C}$. The modulus of elasticity for steel is 210 GPa.

What is most nearly the axial force in the bar if the temperature is raised to 40°C?

(A) 92 kN
(B) 110 kN
(C) 130 kN
(D) 150 kN
SOLUTIONS

1. The polar moment of inertia is

\[ J = \frac{\pi r^4}{2} = \frac{\pi}{2} \left( \frac{0.15 \text{ m}}{2} \right)^4 \]

\[ = 4.97 \times 10^{-5} \text{ m}^4 \]

The shear stress is

\[ \tau = \frac{T r}{J} = \frac{13500 \text{ N.m} \left( \frac{0.15 \text{ m}}{2} \right)}{4.97 \times 10^{-5} \text{ m}^4} \]

\[ = 20.37 \times 10^6 \text{ Pa} \quad (20 \text{ MPa}) \]

The answer is (A).

2. Changes in temperature affect each linear dimension.

\[ \delta_{\text{width}} = \alpha L (T - T_o) \]

\[ = \left(8.8 \times 10^{-6} \frac{1}{\text{C}} \right) (1.2 \text{ m})(50^\circ \text{C} - 0^\circ \text{C}) \]

\[ = 0.000528 \text{ m} \]

\[ \delta_{\text{height}} = \left(8.8 \times 10^{-6} \frac{1}{\text{C}} \right) (2 \text{ m})(50^\circ \text{C} - 0^\circ \text{C}) \]

\[ = 0.00088 \text{ m} \]

\[ A_{\text{initial}} = (2 \text{ m})(1.2 \text{ m}) = 2.4 \text{ m}^2 \]

\[ A_{\text{final}} = (2 \text{ m} + 0.00088 \text{ m}) \times (1.2 \text{ m} + 0.000528 \text{ m}) \]

\[ = 2.40211 \text{ m}^2 \]

\[ \Delta A = A_{\text{final}} - A_{\text{initial}} \]

\[ = 2.40211 \text{ m}^2 - 2.4 \text{ m}^2 \]

\[ = 0.00211 \text{ m}^2 \quad (0.0021 \text{ m}^2) \]

Alternative Solution

The area coefficient of thermal expansion is, for all practical purposes, equal to \(2\alpha\).

The change in area is

\[ \Delta A = 2\alpha A_o \Delta T \]

\[ = (2) \left(8.8 \times 10^{-6} \frac{1}{\text{C}} \right) (2.4 \text{ m}^2)(50^\circ \text{C} - 0^\circ \text{C}) \]

\[ = 0.00211 \text{ m}^2 \quad (0.0021 \text{ m}^2) \]

The answer is (C).

3. Determine whether the tank is thin-walled or thick-walled.

\[ \frac{t}{r} = \frac{12.5 \text{ mm}}{\frac{3.5 \text{ m}}{2}} \left( \frac{1000 \text{ mm/m}}{} \right) = 0.007 < 0.1 \]

Use formulas for thin-walled cylindrical tanks. The pressure is

\[ p = \rho gh \]

\[ = \left(1198 \frac{\text{kg}}{\text{m}^3} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right) (5 \text{ m} - 0.65 \text{ m}) \]

\[ = 51123 \text{ Pa} \]

The hoop stress is

\[ \sigma_t = \frac{pr}{t} = \frac{pd}{2t} = \frac{(51123 \text{ Pa})(3.5 \text{ m})}{(2) \left( \frac{12.5 \text{ mm}}{1000 \text{ mm/m}} \right)} \]

\[ = 7.157 \times 10^6 \text{ Pa} \quad (7.2 \text{ MPa}) \]

The answer is (C).

4. Convert the twist angle to radians.

\[ \phi = (1.5^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = 0.026 \text{ rad} \]

Calculate the polar moment of inertia, \(J\).

\[ r_1 = 15 \text{ mm} \quad (0.015 \text{ m}) \]

\[ r_2 = 25 \text{ mm} \quad (0.025 \text{ m}) \]

\[ J = \frac{\pi}{2} (r_o^4 - r_i^4) = \frac{\pi}{2} \left( (0.025 \text{ m})^4 - (0.015 \text{ m})^4 \right) \]

\[ = 5.34 \times 10^{-7} \text{ m}^4 \]

The torque is

\[ T = \frac{\phi G J}{L} \]

\[ = (0.026 \text{ rad})(80 \text{ GPa}) \left( \frac{10^9 \text{ Pa}}{\text{GPa}} \right) \frac{(5.34 \times 10^{-7} \text{ m}^4)}{1.0 \text{ m}} \]

\[ = 1119 \text{ N.m} \quad (1100 \text{ N.m}) \]

The answer is (D).

5. Since the shear stress is largest at the outer diameter, the maximum torque is found using this radius. For an annular region,

\[ J = \frac{\pi}{2} (r_o^4 - r_i^4) = \frac{\pi}{2} \left( (0.025 \text{ m})^4 - (0.015 \text{ m})^4 \right) \]

\[ = 5.34 \times 10^{-7} \text{ m}^4 \]
The torque is
\[
T_{\text{max}} = \frac{\tau J}{r_2} = \frac{(110 \text{ MPa}) \left( \frac{10^6 \text{ Pa}}{\text{MPa}} \right) (5.34 \times 10^{-7} \text{ m}^4)}{0.025 \text{ m}}
= 2349.9 \text{ N} \cdot \text{m} \quad (2300 \text{ N} \cdot \text{m})
\]

The answer is (C).

6. Convert degrees to radians.
\[
\phi = (4.5^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right)
= 7.854 \times 10^{-2} \text{ rad}
\]

The polar moment of inertia is
\[
J = \frac{\pi r^4}{2} = \frac{(\pi)}{2} \left( \frac{25 \text{ mm}}{100 \text{ mm/m}} \right)^4
= 3.83 \times 10^{-8} \text{ m}^4
\]

Rearrange the twist angle equation to solve for torque.
\[
T = \frac{\phi GJ}{L}
= \frac{(7.854 \times 10^{-2} \text{ rad})(2.8 \times 10^{10} \text{ Pa})(3.83 \times 10^{-8} \text{ m}^4)}{50 \text{ cm}}
= 168.7 \text{ N} \cdot \text{m} \quad (170 \text{ N} \cdot \text{m})
\]

The answer is (D).

7. The elongation due to temperature change is
\[
\delta = \alpha L (T_2 - T_1)
= (9.4 \times 10^{-6} \text{ mm/}^\circ \text{C})(1000 \text{ mm})(40^\circ \text{C} - 10^\circ \text{C})
= 0.282 \text{ mm}
\]

Rearrange the elongation equation to solve for force.
\[
F = \frac{\delta EA}{L}
= \frac{(0.282 \text{ mm})(200 \text{ GPa}) \left( \frac{10^6 \text{ kPa}}{\text{GPa}} \right) (2600 \text{ mm}^2)}{(1 \text{ m})(1000 \text{ mm/m})^3}
= 146.6 \text{ kN} \quad (150 \text{ kN})
\]

The answer is (D).

8. Maximum shear stress occurs at the outer surface. The shear is
\[
\tau = \frac{T_r}{J} = \frac{T (\frac{d}{2})}{\frac{\pi d^4}{32}} = \frac{(280 \text{ N} \cdot \text{m}) \left( \frac{3 \text{ m}}{2} \right)}{(\frac{\pi}{32})(3 \text{ m})^4}
= 52.8 \text{ Pa} \quad (53 \text{ Pa})
\]

The answer is (D).

9. The thermal strain is
\[
\varepsilon_t = \alpha \Delta T = (11.7 \times 10^{-6} \text{ } \frac{1}{\circ \text{C}})(50^\circ \text{C})
= 0.000585 \text{ m/m}
\]

The thermal stress is
\[
\sigma_t = E \varepsilon_t = (210 \text{ GPa}) \left( \frac{10^9 \text{ Pa}}{\text{GPa}} \right) (0.000585 \text{ m/m})
= 1.2285 \times 10^8 \text{ Pa}
\]

(This is less than the yield strength of steel.)

The compressive force in the rod is
\[
F = \sigma A
= (1.2285 \times 10^8 \text{ Pa})\pi \left( \frac{12.5 \text{ mm}}{2} \right)^2 \left( \frac{1000 \text{ mm/m}}{2} \right)
= 15.076 \text{ kN} \quad (15 \text{ kN})
\]

The answer is (B).

10. The total change in length is
\[
\delta_t = \alpha L_{\text{initial}} (T - T_o)
= \left( 11 \times 10^{-6} \text{ } \frac{1}{\circ \text{C}} \right)(10 \text{ km})(50^\circ \text{C} - 20^\circ \text{C})
= 0.0033 \text{ km}
\]

Add the change in length to the initial length.
\[
L = L_{\text{initial}} + \delta_t
= 10 \text{ km} + 0.0033 \text{ km}
= 10.0033 \text{ km}
\]

The answer is (C).

11. Tanks under external pressure fail by buckling (i.e., collapse), not by yielding. They should not be designed using the simplistic formulas commonly used for thin-walled tanks under internal pressure.

The answer is (D).