Problem 1

Kelly just won the state lottery. The $2,000,000 jackpot will be paid in 20 annual installments of $100,000. The first installment is given to Kelly immediately. The interest rate is 6% compounded yearly. The present worth of Kelly’s lottery winnings (at the time of receiving the first installment) is most nearly

(A) $1,120,000
(B) $1,150,000
(C) $1,220,000
(D) $1,900,000

Solution

Use the uniform series present worth factor to find the present worth of the last 19 installments at the time the first installment is made. Add this value to the first installment to determine the total present worth of Kelly’s lottery winnings.

\[
A = \text{uniform series of end of compounding period cash flows} \\
= 100,000 \\
i = \text{annual interest rate} = 6\% \\
n = \text{number of compounding periods} = 19 \text{ years} \\
P = \text{present equivalent value of a cash flow or series of cash flows} \\
= 100,000 + (100,000)(P/A, 6\%, 19) \\
= 100,000 + (100,000)\left(\frac{(1+0.06)^{19} - 1}{(0.06)(1+0.06)^{19}}\right) \\
= 100,000 + (100,000)(11.1581) \\
= 1,215,810 \quad ($1,220,000)
\]

The answer is C.

Problem 2

Joe wants to be a millionaire. To achieve this goal, at the end of the year he invests $5000 each year into an account that pays 10% interest, compounded yearly. The amount of time it will take Joe to reach his goal of becoming a millionaire is most nearly

(A) 20 yr 
(B) 30 yr 
(C) 40 yr 
(D) 180 yr

Solution

Use the uniform series compound amount factor to relate the future worth of the account to the yearly deposits.

\[
A = \text{uniform series of end of compounding period cash flows} \\
= 5000 \\
i = \text{annual interest rate} = 10\% \\
n = \text{number of compounding periods} = \text{number of years} \\
F = \text{future equivalent value of a cash flow or series of cash flows} \\
= 1,000,000 \\
1,000,000 = (5000)(F/A, 10\%, n) \\
= (5000)\left(\frac{(1+0.1)^n - 1}{0.1}\right) \\
21 = 1.1^n \\
\log 21 = n \log 1.1 \\
n = \frac{\log 21}{\log 1.1} = 31.94 \text{ yr} \quad (30 \text{ yr})
\]

The answer is B.
PROBLEMS

1. Jane is planning for her retirement. Each month she places $200 in an account that pays 12% nominal interest, compounded monthly. She made the first deposit of $200 on January 31, 1997. The last $200 deposit will be made on December 31, 2016. If the interest rate remains constant and all deposits are made as planned, the amount in Jane’s retirement account on January 1, 2017 is most nearly

(A) $60,000  
(B) $155,000  
(C) $173,000  
(D) $198,000

2. Dawn purchased a $10,000 car. She put $2000 down and financed the $8000 balance. The interest rate is 9% nominal, compounded monthly, and the loan is to be repaid in equal monthly installments over the next four years. Dawn’s monthly car payment is most nearly

(A) $167  
(B) $172  
(C) $188  
(D) $200

3. James is a major prizewinner in a sweepstakes. He has the option of either receiving a single check for $125,000 now or receiving a check for $50,000 each year for three years. (James would be given the first $50,000 check now.) At what interest rate would James most nearly have to invest his winnings for him to be indifferent as to how he receives his winnings?

(A) 16%  
(B) 20%  
(C) 22%  
(D) 23%

4. In the design of an automobile radiator, an engineer has a choice of using either a brass-copper alloy casting or a plastic molding. Either material provides the same service. However, the brass-copper alloy casting has a mass of 25 lbm, compared with 16 lbm for the plastic molding. Every pound of extra mass in the automobile has been assigned a penalty of $4 to account for increased fuel consumption during the lifecycle of the car. The brass-copper alloy casting costs $3.35 per pound, whereas the plastic molding costs $7.40 per pound. Machining costs per casting are $6.00 for the brass-copper alloy. Which material should the engineer select, and what is the difference in unit cost?

(A) brass-copper alloy, savings = $1.35/radiator  
(B) brass-copper alloy, savings = $6.00/radiator  
(C) plastic, savings = $7.35/radiator  
(D) plastic, savings = $8.50/radiator

5. A process engineer is trying to decide on the type of tool material to use for a machining operation. Relevant data for the alternatives are as follows.

<table>
<thead>
<tr>
<th>tool material A</th>
<th>tool material B</th>
</tr>
</thead>
<tbody>
<tr>
<td>tool cost</td>
<td>$100</td>
</tr>
<tr>
<td>production rate</td>
<td>100 pieces/hr</td>
</tr>
<tr>
<td>tool life</td>
<td>50 hr</td>
</tr>
</tbody>
</table>

It takes 1 hour to replace a worn tool. Labor costs $18 per hour, and a total of 15,000 pieces are needed. What tool material should be selected, and what are the expected savings?

(A) material A, save $900  
(B) material B, save $1000  
(C) material A, save $1000  
(D) material B, save $3100

6. A project engineer is deciding on the most cost-effective duration for a new project. The direct costs of the project are expected to vary indirectly with project duration, while indirect costs vary directly with the square of the project duration. The total project cost is represented by the following equation.

\[
\text{project cost} = 5000 + \frac{4000}{T} + \frac{250T^2}{\text{mo}^2}
\]

where \(T\) is the project duration in months

For what duration should the project be planned, and what is most nearly the expected project cost?

(A) 3 mo, $600  
(B) 2 mo, $3000  
(C) 2 mo, $8000  
(D) 3 mo, $8600
1. Use the uniform series compound amount factor to find the future worth of the deposits.

\[ F = A(F/A, i, n) \]

\[ A = \text{uniform series of end of compounding period cash flows} = \$200 \]

\[ i = \text{interest rate per compounding period} = 12\% \]

\[ n = \text{number of compounding periods} = (20 \text{ years})(12 \text{ compounding periods per year}) = 240 \text{ compounding periods} \]

\[ F = \$200 \times (F/A, 1\%, 240) \]

\[ = \$200 \left( \frac{(1 + 0.01)^{240} - 1}{0.01} \right) \]

\[ = \$197,851 \quad (\$198,000) \]

The answer is D.

2. Use the capital recovery factor to find the monthly car payment.

\[ A = \text{uniform series of end of period cash flows} = P(A/P, i, n) \]

\[ P = \text{present equivalent value of a cash flow or series of cash flows (loan amount)} = \$8000 \]

\[ i = \text{interest rate per compounding period} = 9\% \]

\[ n = \text{number of compounding periods} = (4 \text{ years})(12 \text{ compounding periods per year}) = 48 \text{ compounding periods} \]

\[ A = \$8000 \times (A/P, 0.75\%, 48) \]

\[ = \$8000 \left( \frac{0.0075(1 + 0.0075)^{48}}{(1 + 0.0075)^{48} - 1} \right) \]

\[ = \$199.20 \quad (\$200) \]

The answer is D.

3. Equate the equivalent worth of the two options and solve for the interest rate.

Option 1, receive $125,000 now \((P_0 = \$125,000)\).

Option 2, receive $50,000 now and $50,000 each year for two years.

Use the uniform series present worth factor.

\[ P_0 = \$50,000 + A(P/A, i, n) \]

\[ A = \$50,000 \]

\[ n = 2 \]

\[ P_0 = \$50,000 + (\$50,000)(P/A, i, 2) \]

\[ = \$50,000 + (\$50,000) \left( \frac{(1 + i)^2 - 1}{i(1 + i)^2} \right) \]

Equate the present worth of the options and solve for the interest rate.

\[ \$125,000 = \$50,000 + (\$50,000) \left( \frac{(1 + i)^2 - 1}{i(1 + i)^2} \right) \]

\[ 1.5 = \left( \frac{(1 + i)^2 - 1}{i(1 + i)^2} \right) = (P/A, i, 2) \]

By trial and error, a range for \(i\) can be determined. Then, use linear interpolation to approximate \(i\).
\[
\frac{25\% - i}{1.44 - 1.5} = \frac{25\% - 20\%}{1.44 - 1.5278} \\
i = 0.1966 \ (20\%)
\]

The answer is B.

4. Cost factor | brass-copper alloy | plastic molding |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>casting</td>
<td>(\text{25 lbm/}$3.35/\text{lbm})</td>
<td>(\text{16 lbm/}$7.40/\text{lbm})</td>
</tr>
<tr>
<td></td>
<td>$83.75</td>
<td>$118.40</td>
</tr>
<tr>
<td>machining</td>
<td>$6.00</td>
<td>0.00</td>
</tr>
<tr>
<td>weight</td>
<td>(\text{25 lbm} - \text{16 lbm}$/4/\text{lbm})</td>
<td></td>
</tr>
<tr>
<td>penalty</td>
<td>$36.00</td>
<td>0.00</td>
</tr>
<tr>
<td>total cost</td>
<td>$125.75</td>
<td>$118.40</td>
</tr>
</tbody>
</table>

The plastic molding should be selected to save \$125.75 - \$118.40 = \$7.35 over the lifecycle of each radiator.

The answer is C.

5. Determine the cost of producing 15,000 pieces for each tool material.

\[
\text{total cost} = (\text{tool cost})(\text{no. of tools needed}) + \left(\frac{\text{labor cost}}{\text{hr}}\right)(\text{hr needed})
\]

\[
\text{tools needed} = \frac{\text{pieces required}}{\left(\frac{\text{pieces}}{\text{hr}}\right) \left(\frac{\text{tool life in hr}}{\text{tool}}\right)}
\]

\[
\text{hours needed} = \text{production time} + \text{tool changing time}
\]

\[
= \frac{\text{pieces required}}{\text{hr}} + \left(\frac{1}{\text{tool}}\right)(\text{no. of tools needed})
\]

For material A,

\[
\text{tools needed} = \frac{15,000 \text{ pieces}}{\left(\frac{100 \text{ pieces}}{\text{hr}}\right) \left(\frac{50 \text{ hr}}{\text{tool}}\right)} = 3 \text{ tools}
\]

\[
\text{hours needed} = \frac{15,000 \text{ pieces}}{100 \text{ pieces/hr}} + \left(\frac{1}{\text{tool}}\right)(3 \text{ tools}) = 153 \text{ hr}
\]

\[
\text{total cost} = \left(\frac{100 \text{ $}}{\text{tool}}\right)(3 \text{ tools}) + \left(\frac{18 \text{ $}}{\text{hr}}\right)(153 \text{ hr}) = \$3054
\]

For material B,

\[
\text{tools needed} = \frac{15,000 \text{ pieces}}{\left(\frac{75 \text{ pieces}}{\text{hr}}\right) \left(\frac{25 \text{ hr}}{\text{tool}}\right)} = 8 \text{ tools}
\]

\[
\text{hours needed} = \frac{15,000 \text{ pieces}}{75 \text{ pieces/hr}} + \left(\frac{1}{\text{tool}}\right)(8 \text{ tools}) = 208 \text{ hr}
\]

\[
\text{total cost} = \left(\frac{30 \text{ $}}{\text{tool}}\right)(8 \text{ tools}) + \left(\frac{18 \text{ $}}{\text{hr}}\right)(208 \text{ hr}) = \$3984
\]

Material A should be selected. The savings are \$3984 - \$3054 = \$930.

The answer is A.

6. To minimize project cost, set the first derivative of the project cost equation (with respect to project length) equal to zero and solve for the project length, \(T\).

\[
\frac{d(\text{project cost})}{dT} = \frac{-4000 \text{ mo}}{T^2} + \frac{500T}{\text{mo}^2} = 0
\]

\[
500T^3 = 4000 \text{ mo}^3
\]

\[
T = \sqrt[3]{8} \text{ mo}^3 = 2 \text{ mo}
\]

To find the expected project cost, substitute \(T = 2 \text{ months}\) into the project cost equation.

\[
\text{project cost} = 5000 + \frac{4000 \text{ mo}}{T} + \frac{250T^2}{\text{mo}^2}
\]

\[
= 5000 + \frac{4000 \text{ mo}}{2 \text{ mo}} + \frac{(250)(2 \text{ mo})^2}{\text{mo}^2}
\]

\[
= \$8000
\]

The answer is C.

7. To calculate the benefit-cost ratio for the new Colorado State Park, first classify the benefits and costs.

\[
\text{benefits} = (\$3 \text{ per person entry fee} + \$15 \text{ per person positive economic impact})(500,000 \text{ people per year}) = \$9,000,000 \text{ per year}
\]

\[
\text{costs} = \$20,000,000 \text{ development cost at } t_0 \text{ and } \$2,000,000 \text{ per year operating expense}
\]

\[
\text{The answer is J.}
\]