1. Strut-and-Tie Models .................................................. 1-1
2. Corbels ................................................................. 1-6
3. Design for Torsion ..................................................... 1-8
4. Practice Problems .................................................... 1-13

1. STRUT-AND-TIE MODELS

Nomenclature

- $A_{cs}$: effective cross-sectional area of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut (in$^2$)
- $A_{nz}$: effective cross-sectional area of the face of a nodal zone (in$^2$)
- $A_{id}$: area of reinforcement in the $i$th layer of reinforcement crossing the strut (in$^2$)
- $A'_i$: area of compression reinforcement (in$^2$)
- $A_{to}$: total area of nonprestressed reinforcement in a tie (in$^2$)
- $A_s$: area of shear reinforcement perpendicular to tension reinforcement within a distance $s$ (in$^2$)
- $A_{sh}$: area of shear reinforcement parallel to tension reinforcement within a distance $s$ (in$^2$)
- $b$: width of member (in)
- $b_w$: web width (in)
- $c$: clear cover to reinforcement (in)
- $C$: compressive force acting on a nodal zone (kips)
- $d$: distance from extreme compression fiber to centroid of longitudinal tension reinforcement (in)
- $d_b$: nominal diameter of bar, wire, or prestressing strand (in)
- $f'_c$: compressive strength of concrete (lbf/in$^2$)
- $f_{ce}$: effective compressive strength of the concrete in a strut (kips/in$^2$)
- $f_s$: stress in compression reinforcement under factored loads (lbf/in$^2$)
- $F_{na}$: nominal compressive strength of a strut (kips)
- $F_{td}$: nominal tensile strength of a tie (kips)
- $h$: depth of member (in)
- $l_a$: anchorage length of a reinforcing bar (in)
- $l_b$: width of bearing plate (in)
- $l_{dh}$: development length in tension of a hooked bar (in)
- $R$: support reaction acting on a nodal zone (kips)
- $s$: stirrup spacing (in)
- $s_i$: spacing of the $i$th layer of reinforcement (in)
- $T$: tension force acting on a nodal zone (kips)
- $w_s$: effective width of strut perpendicular to the axis of the strut (in)
- $w_t$: effective width of concrete concentric with a tie (in)

Symbols

- $a_i$: angle between the strut and the bars in the $i$th layer of reinforcement crossing the strut (deg)
- $b$: factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone
- $b_s$: factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut
- $\theta$: angle between the axis of a strut and a tension chord (deg)
- $\lambda$: correction factor related to unit weight of concrete
- $\rho$: reinforcement ratio
- $\phi$: strength reduction factor

A strut-and-tie model consists of the application of an analogous truss model to regions of a member in which the stress distribution is nonuniform and the normal beam theory does not apply. An example of this situation is a deep beam, which may be designed using nonlinear analysis methods or by the strut-and-tie analogy method given in ACI 318.\(^1\) A deep beam is defined in ACI Sec. 9.9.1.1 as a beam in which the ratio of clear span to overall depth does not exceed four, or in which the shear span to depth ratio does not exceed two.

As shown in Fig. 1.1, a discontinuity, or D region, occurs at a change in the geometry of a member and at concentrated loads and reactions. The D region extends a distance equal to the overall depth of the member from the location of the change in geometry or the point of application of a load or reaction. Outside of the D region, beam theory is applicable, and this region is known as a beam, or B, region.

\(^1\)American Concrete Institute, 2014. (See References and Codes.)
4. PRACTICE PROBLEMS

For Prob. 1 through Prob. 3, refer to the illustration shown. The reinforced concrete beam has a width of 14 in and a concrete compressive strength of 5000 lbf/in². The factored applied force of 200 kips includes an allowance for the self-weight of the beam.

1. Using the equivalent strut-and-tie model indicated, what is most nearly the area of grade 60 tension reinforcement required?
   (A) 1.78 in²
   (B) 2.67 in²
   (C) 3.33 in²
   (D) 3.56 in²

2. Is the equivalent, unreinforced bottle-shaped concrete strut at the right support compliant with the requirements of ACI Chap. 23?
   (A) No, the design compressive strength of the concrete is 1.91 kips/in².
   (B) No, the design compressive strength of the concrete is 2.55 kips/in².
   (C) Yes, the design compressive strength of the concrete is 2.29 kips/in².
   (D) Yes, the design compressive strength of the concrete is 2.55 kips/in².

3. Is the equivalent nodal zone at the right support compliant with the requirements of ACI Chap. 23?
   (A) No, the design compressive strength is 1.90 kips/in².
   (B) No, the design compressive strength is 2.10 kips/in².
   (C) Yes, the design compressive strength is 2.55 kips/in².
   (D) Yes, the design compressive strength is 3.40 kips/in².

4. What is most nearly the maximum factored torsion that the beam can support?
   (A) 340 in-kips
   (B) 370 in-kips
   (C) 420 in-kips
   (D) 560 in-kips

5. What is most nearly the required area of longitudinal reinforcement for the beam?
   (A) 1.40 in²/arm
   (B) 1.60 in²/arm
   (C) 1.90 in²/arm
   (D) 2.10 in²/arm

Problem 4 and Prob. 5 refer to the reinforced concrete beam shown. The concrete strength is 3000 lbf/in², and all reinforcement is grade 60. The concrete section is adequate to support the applied shear force.
1. For the idealized strut-and-tie model shown in the illustration, the angle between the strut and tie at the right support is

\[ \theta = \tan^{-1}\left(\frac{30 \text{ in}}{60 \text{ in}}\right) = 26.6^\circ > 25^\circ \text{ [satisfies ACI Sec. A.2.5]} \]

The reaction at the right support is

\[ R = \frac{(200 \text{ kips})(40 \text{ in})}{100 \text{ in}} = 80 \text{ kips} \]

The equivalent tie force is determined from the strut-and-tie model as

\[ T = \frac{(80 \text{ kips})(60 \text{ in})}{30 \text{ in}} = 160 \text{ kips} \]

The strength reduction factor is given by ACI Sec. 21.2.1 as

\[ \phi = 0.75 \]

The necessary reinforcement area is

\[ A_{ts} = \frac{T}{\phi f_y} = \frac{160 \text{ kips}}{(0.75)(60 \text{ kips/in}^2)} = 3.56 \text{ in}^2 \]

Use five no. 8 bars, which gives

\[ A = 3.95 \text{ in}^2 > A_{ts} \text{ [satisfactory]} \]

The answer is (D).

2. The dimensions of the nodal zone are the equivalent tie width, \( w_t \), and the width of the equivalent concrete strut, \( w_s \).

\[ w_t = d_h + 2c = 1 \text{ in} + (2)(2 \text{ in}) = 5 \text{ in} \]

\[ w_s = \frac{w_t}{\cos \theta} = \frac{5 \text{ in}}{\cos 26.6^\circ} = 5.59 \text{ in} \]

The stress in the equivalent tie is

\[ f_T = \frac{T}{bw_t} = \frac{160 \text{ kips}}{(14 \text{ in})(5 \text{ in})} = 2.29 \text{ kips/in}^2 \]

For a hydrostatic nodal zone, the stress in the equivalent concrete strut is

\[ f_c = f_T = 2.29 \text{ kips/in}^2 \]

For normal weight concrete and an unconfined, bottle-shaped strut, the effective compressive strength of the concrete in the strut is given by ACI Eq. 23.4.3 as

\[ f_{cc} = 0.85f_c' = (0.85)(0.6)(1.0)\left(\frac{5 \text{ kips}}{\text{in}^2}\right) = 2.55 \text{ kips/in}^2 \]

The design compressive stress is

\[ \phi f_{cc} = (0.75)(2.55 \text{ kips/in}^2) = 1.91 \text{ kips/in}^2 < f_C \text{ [unsatisfactory]} \]

The answer is (A).

3. The nominal compressive strength of a nodal zone anchoring one layer of reinforcing bars without confining reinforcement is given by ACI Eq. 23.9.2 as

\[ f_{cc} = 0.85f_c' = (0.85)(0.8)\left(\frac{5 \text{ kips}}{\text{in}^2}\right) = 3.40 \text{ kips/in}^2 \]

The design compressive stress is

\[ \phi f_{cc} = (0.75)(3.40 \text{ kips/in}^2) = 2.55 \text{ kips/in}^2 > f_C \text{ [satisfactory]} \]

The answer is (C).

4. The area enclosed by the centerline of the stirrups is

\[ A_{oh} = (19 \text{ in} - 3 \text{ in} - 0.5 \text{ in})(12 \text{ in} - 3 \text{ in} - 0.5 \text{ in}) = 131.75 \text{ in}^2 \]
In accordance with TMS 402 Eq. 8-25, the shear stress resisted by the masonry and the shear stress resisted by the shear reinforcement are additive to give a combined allowable shear stress of

\[ F_v = F_{vm} + F_{vs} \]

The allowable shear stress, when \( M/V_d \leq 0.25 \), is limited by TMS 402 Eq. 8-26 to

\[ F_v \leq 3\sqrt{f_m} \]

The allowable shear stress, when \( M/V_d \geq 1.0 \), is limited by TMS 402 Eq. 8-27 to

\[ F_v \leq 2\sqrt{f_m} \]

To simplify the procedure, TMS 402 Comm. Sec. 8.3.5.1.2 permits \( M/V_d \) to be 1.0 in TMS 402 Eq. 2-29 and TMS 402 Sec. 8.3.5.1.2.

Example 6.6

The nominal 8 in, solid grouted concrete block masonry beam shown in Illustration for Example 6.6 has an effective depth, \( d \), of 45 in and an overall depth of 48 in. The masonry has a compressive strength of 1500 lbf/in\(^2\). The flexural reinforcement consists of one no. 8 grade 60 bar. For the loading indicated, determine whether shear stirrups are required at the ends of the beam, and if so, the amount required. Use no. 4 grade 60 reinforcement.

Solution

The critical section for shear for a beam, in accordance with TMS 402 Sec. 8.3.5.4, occurs at a distance of \( d/2 \) from the face of the support. The shear at the critical section is derived in Ex. 6.1 as

\[ V = 21 \text{ kips} \]

The shear stress at the critical section is given by TMS 402 Eq. 8-24 as

\[ f_v = \frac{V}{bd_v} \]

\[ = \frac{21 \text{ kips}}{1000 \text{ lbf/kip}} \]

\[ = \frac{21 \text{ kips}}{7.63 \text{ in}} \]

\[ = 26.21 \text{ lbf/in}^2 \]

Assuming \( M/V_d = 1.0 \), the maximum permitted shear stress with shear reinforcement provided is limited by TMS 402 Eq. 8-27 to

\[ F_v = 2\sqrt{f_m} \]

\[ = 2\sqrt{1500 \text{ lbf/in}^2} \]

\[ = 77.46 \text{ lbf/in}^2 \]

\[ > f_v \quad \text{[satisfactory]} \]

The allowable shear stress resisted by the masonry is given by TMS 402 Eq. 8-29 as

\[ F_v = \frac{F_{vm}}{A_n} \]

\[ = \frac{57.3 \text{ lbf/in}^2}{43.57 \text{ lbf/in}^2} \]

\[ = 43.57 \text{ lbf/in}^2 \]

\[ < f_v \quad \text{[shear reinforcement is required]} \]

The residual shear stress is

\[ f_{vr} = f_v - F_{vm} \]

\[ = 57.3 \text{ lbf/in}^2 - 43.57 \text{ lbf/in}^2 \]

\[ = 13.73 \text{ lbf/in}^2 \]

Provide a no. 4 bar (0.20 in\(^2\)) at 16 in centers. This provides a shear stress resistance, as specified by TMS 402 Eq. 8-30, of

\[ F_{vs} = 0.5 \left( \frac{A_v F_{sd_v}}{A_{ns}} \right) \]

\[ = 0.5 \left( \frac{0.20 \text{ in}^2(32,000 \text{ lbf/in}^2)(48 \text{ in})}{(7.63 \text{ in})(48 \text{ in})(16 \text{ in})} \right) \]

\[ = 26.21 \text{ lbf/in}^2 \]

The combined allowable shear stress is given by TMS 402 Eq. 8-25 as

\[ F_v = F_{vm} + F_{vs} \]

\[ = 43.57 \text{ lbf/in}^2 + 26.21 \text{ lbf/in}^2 \]

\[ = 69.78 \text{ lbf/in}^2 \]

\[ > f_v = 57.3 \text{ lbf/in}^2 \quad \text{[satisfactory]} \]