ENGINEER - IN - TRAINING REFERENCE MANUAL

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Michael R. Lindeburg, P.E.

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17 FLUID DYNAMICS

Nomenclature

a	length	ft	m
a	speed of sound	ft/sec	m/s
A	area	ft^2	m^2
C	Hazen-Williams coefficient	–	–
C	coefficient	–	–
D	diameter	ft	m
E	specific energy	ft-lbf/slug	J/kg
E	specific energy (customary U.S.)	ft-lbf/lbm	n.a.
E	bulk modulus	lbf/ft^2	Pa
f	Darcy friction factor	–	–
f	fraction split	–	–
F	force	lbf	N
Fr	Froude number	–	–
g	gravitational acceleration	ft/sec^2	m/s^2
g_c	gravitational conversion constant (32.2)	$lbm\text{-}ft/lbf\text{-}sec^2$	n.a.
G	mass flow rate per unit area	$slug/ft^2\text{-}sec$	$kg/m^2{\cdot}s$
G	mass flow rate per unit area (customary U.S.)	$lbm/ft^2\text{-}sec$	n.a.
h	height or head	ft	m
HP	horsepower	hp	n.a.
I	impulse	lbf-sec	N·s
k	ratio of specific heats	–	–
K	minor loss coefficient	–	–
l	length	ft	m
L	length	ft	m
m	mass	slug	kg
m	mass (customary U.S.)	lbm	n.a.
$\dot{m}$	mass flow rate	slug/sec	kg/s
$\dot{m}$	mass flow rate (customary U.S.)	lbm/sec	n.a.
MW	molecular weight	lbm/lbmole	kg/kmol
n	Manning roughness constant	–	–
p	pressure	lbf/ft^2	Pa
P	power	ft-lbf/sec	W
P	momentum	slug-ft/sec	kg·m/s
P	momentum (customary U.S.)	lbm-ft/sec	n.a.
r	radius	ft	m

rpm	rotational speed	rev/min	rev/min
R	resultant force	lbf	N
R^*	universal gas constant	ft-lbf/lbmole-°R	J/kmol·K
Re	Reynolds number	–	–
t	time	sec	s
t	thickness	ft	m
T	absolute temperature	°R	K
u	x-component of velocity	ft/sec	m/s
v	y-component of velocity	ft/sec	m/s
v	velocity	ft/sec	m/s
V	volume	ft^3	m^3
$\dot{V}$	volumetric flow rate	ft^3/sec	m^3/s
W	work	ft-lbf	J
We	Weber number	–	–
WHP	water horsepower	hp	n.a.
x	x-coordinate of position	ft	m
y	y-coordinate of position	ft	m
Y	expansion factor	–	–
z	elevation	ft	m

Symbols

β	diameter ratio	–	–
Γ	circulation	ft^2/sec	m^2/s
ϵ	specific roughness	ft	m
η	efficiency	–	–
θ	angle	degrees	degrees
μ	absolute viscosity	lbf-sec/ft^2	Pa·s
ν	kinematic viscosity	ft^2/sec	m^2/s
ρ	density	slugs/ft^3	kg/m^3
ρ	density (customary U.S.)	lbm/ft^3	n.a.
σ	surface tension	lbf/ft	N/m
τ	shear stress	lbf/ft^2	Pa
ϕ	angle	degrees	degrees
Φ	stream potential	–	–
psi	sphericity	–	–
Ψ	stream function	–	–
ω	angular velocity	rad/sec	rad/s

Subscripts

A	added (by pump)
b	blade or buoyant
c	contraction
d	discharge
D	drag
e	equivalent
E	extracted (by turbine)
f	friction or flow
I	instrument
L	lift
m	minor, model, or manometer fluid
o	orifice
p	pressure or prototype
r	ratio
s	static
t	total, tank, or theoretical
v	velocity
va	velocity of approach
z	potential

1 HYDRAULICS AND HYDRODYNAMICS

This chapter investigates fluid moving through pipes, measurements with venturis and orifices, and other motion-related topics such as model theory, lift and drag, and pumps. In a strict interpretation, any fluid-related phenomenon that isn't hydro*statics* should be hydro*dynamics*. However, tradition has separated the study of moving fluids into the fields of hydraulics and hydrodynamics.

In a general sense, *hydraulics* is the study of the practical laws of fluid flow and resistance in pipes and open channels. Hydraulic formulas are often developed from experimentation, empirical factors, and curve fitting, without an attempt to justify why the fluid behaves the way it does.

On the other hand, *hydrodynamics* is the study of fluid behavior based on theoretical considerations. Hydrodynamicists start with Newton's laws of motion and try to develop models of fluid behavior. Models developed in this manner are complicated greatly by the inclusion of viscous friction and compressibility. Therefore, hydrodynamic models assume a perfect fluid with constant density and zero viscosity. The conclusions reached by hydrodynamicists can differ greatly from those reached by hydraulicians.[1]

2 CONSERVATION OF MASS

Fluid mass is always conserved in fluid systems, regardless of the pipeline complexity, orientation of the flow, and which fluid is flowing. This single concept is often sufficient to solve simple fluid problems.

$$\dot{m}_1 = \dot{m}_2 \qquad 17.1$$

When applied to fluid flow, the conservation of mass law is known as the *continuity equation*.

$$\rho_1 A_1 \mathrm{v}_1 = \rho_2 A_2 \mathrm{v}_2 \qquad 17.2$$

If the fluid is incompressible, then $\rho_1 = \rho_2$.

$$A_1 \mathrm{v}_1 = A_2 \mathrm{v}_2 \qquad 17.3$$

$$\dot{V}_1 = \dot{V}_2 \qquad 17.4$$

3 STREAM POTENTIAL AND STREAM FUNCTION

An application of hydrodynamic theory is the derivation of the stream function from stream potential. The *stream potential function* (*velocity potential function*), Φ, is the algebraic sum of the component velocity potential functions.[2,3]

$$\Phi = \Phi_x(x,y) + \Phi_y(x,y) \qquad 17.5$$

[1]Perhaps the most disparate conclusion is *D'Alembert's paradox*. In 1744, D'Alembert derived theoretical results "proving" that there is no resistance to bodies moving through an ideal (nonviscous) fluid.

[2]The two-dimensional derivation of the stream function can be extended to three dimensions, if necessary.

[3]The stream function can also be expressed in the cylindrical coordinate system.

The velocity component of the resultant in the x direction is

$$u = \frac{\partial \Phi}{\partial x} \qquad 17.6$$

The velocity component of the resultant in the y direction is

$$v = \frac{\partial \Phi}{\partial y} \qquad 17.7$$

The total derivative of the stream potential function is

$$\begin{aligned} d\Phi &= \frac{\partial \Phi}{\partial x}dx + \frac{\partial \Phi}{\partial y}dy \\ &= u\,dx + v\,dy \qquad 17.8 \end{aligned}$$

An *equipotential line* is a line along which the function Φ is constant (e.g., $d\Phi = 0$). The slope of the equipotential line is derived from Eq. 17.8.

$$\left.\frac{dy}{dx}\right|_{\text{equipotential}} = -\frac{u}{v} \qquad 17.9$$

For flow through a porous, permeable medium, pressure will be constant along equipotential lines (i.e., along lines of constant Φ). However, for an ideal, non-viscous fluid flowing in a frictionless environment, Φ has no physical significance. Even though Φ has a theoretical basis, it does not coincide with any measurable physical quantity.

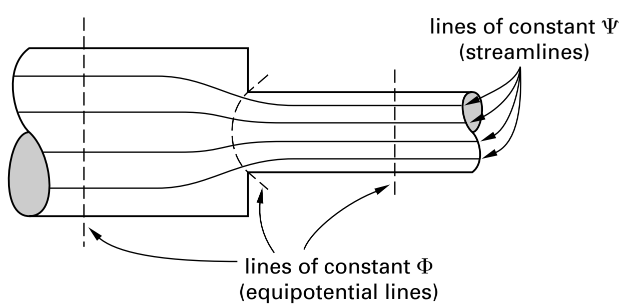


Figure 17.1 Equipotential Lines and Streamlines

The *stream function (Lagrange stream function)*, $\Psi(x, y)$, defines the direction of flow at a point.

$$u = \frac{\partial \Psi}{\partial y} \qquad 17.10$$

$$v = -\frac{\partial \Psi}{\partial x} \qquad 17.11$$

The stream function can also be written in total derivative form.

$$\begin{aligned} d\Psi &= \frac{\partial \Psi}{\partial x}dx + \frac{\partial \Psi}{\partial y}dy \\ &= -v\,dx + u\,dy \qquad 17.12 \end{aligned}$$

The stream function, $\Psi(x, y)$, satisfies Eq. 17.12. For a given streamline, $d\Psi = 0$, and each streamline is a line representing a constant value of Ψ. A streamline is perpendicular to an equipotential line.

$$\left.\frac{dy}{dx}\right|_{\text{streamline}} = \frac{v}{u} \qquad 17.13$$

Example 17.1

The stream potential function for water flowing through a particular valve is

$$\Phi = 3xy - 2y$$

What is the stream function, Ψ?

(solution)

First, work with Φ to obtain u and v.

$$\begin{aligned} u &= \frac{\partial \Phi}{\partial x} = \frac{\partial(3xy - 2y)}{\partial x} = 3y \\ v &= \frac{\partial \Phi}{\partial y} = 3x - 2 \end{aligned}$$

u and v are also related to the stream function, Ψ. From Eq. 17.10,

$$\begin{aligned} u &= \frac{\partial \Psi}{\partial y} \\ \partial \Psi &= u\,\partial y \\ \Psi &= \int 3y\,dy = \frac{3}{2}y^2 + \text{ some function of } x + C_1 \end{aligned}$$

Similarly, from Eq. 17.11,

$$\begin{aligned} v &= -\frac{\partial \Psi}{\partial x} \\ \partial \Psi &= -v\,\partial x \\ \Psi &= -\int (3x - 2)\,dx \\ &= 2x - \frac{3}{2}x^2 + \text{some function of } y + C_2 \end{aligned}$$

Ψ is found by superposition of these two results.

$$\Psi = \frac{3}{2}y^2 + 2x - \frac{3}{2}x^2 + C$$

55 SURFACE TENSION FORCES DOMINATE

Table 17.9 lists some of the cases where surface tension is the predominant force. Such cases can be handled by equating the Weber numbers, We, of the model and prototype.[42] (The *Weber number* is the ratio of inertial force to surface tension.)

$$\text{We} = \frac{\text{v}^2 L\rho}{\sigma} \qquad 17.180$$

$$\text{We}_m = \text{We}_p \qquad 17.181$$

Table 17.9
Cases with Weber Number Similarity

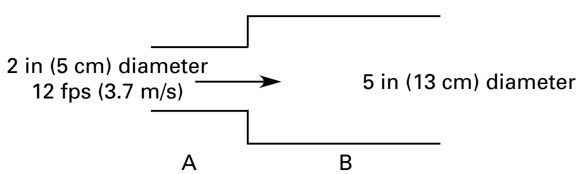
waves
droplets
bubbles
air entrainment

PRACTICE PROBLEMS

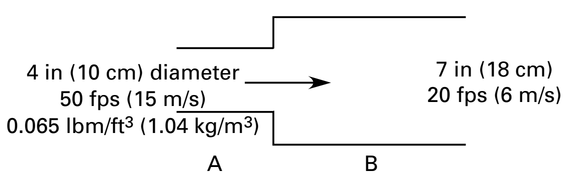
(Use $g = 32.2\,\text{ft/sec}^2$ or $9.81\,\text{m/s}^2$ unless told to do otherwise in the problem.)

Conservation of Mass

1. What is the velocity at point B if water is flowing?



2. What is the density at point B?



Friction Head Loss

3. What is the friction loss per 100 feet (10 m) of 12-inch-diameter (30-cm-diameter) concrete duct if air with an average temperature of 100°F (40°C) flows at 14 ft/sec (4.3 m/s)?

4. What is the head loss for 1000 feet (100 m) of 2-inch (5-cm) galvanized iron pipe (specific roughness of 0.0005 ft, 0.00015 m) if the velocity is 4 ft/sec (1 m/s) and the fluid is 60°F (15°C) water?

5. Points A and B are 3000 feet (900 m) apart along a new 6-inch (15-cm) steel pipe. B is 60 feet (20 m) above A. 750 gal/min (0.047 m^3/s) of fuel oil with a specific gravity of 0.9 flow in the pipe. The flow direction is from A to B. The friction factor is 0.03. What pressure must be maintained at A if the pressure at B is to be 50 psig (350 kPa, gage)?

Minor Losses

6. What is the head loss for a 90° 3-inch-diameter (7.5-cm-diameter) screwed elbow ($K = 0.80$) with water flowing at 15 ft/sec (4.5 m/s)?

7. A smooth 12-inch (30-cm) steel pipe 300 feet (90 m) long has a flush entrance ($K = 0.50$) and a submerged discharge. 70°F (21°C) water flows at 10 ft/sec (3 m/s). What is the total head loss?

8. 70°F (21°C) water flows at 12 ft/sec (3.7 m/s) in a 150 foot (45 m) length of 4-inch (10 cm) steel pipe through an open globe valve and one regular elbow. What is the friction loss?

Pumping Power

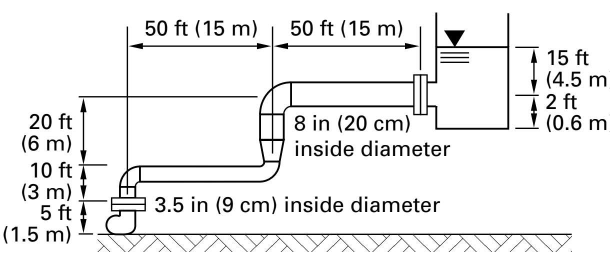
9. A 3-inch-diameter (7.5-cm-diameter) pipe 2000 feet (600 m) long with friction factor of 0.020 carries water from a reservoir and discharges freely at a point 100 feet (30 m) below the reservoir's surface level. Find the pump power required to double the gravity flow.

10. Brine (SG = 1.2) flows through a 2000 gal/min (0.1 m^3/s) pump. The pump outlet is 12-inch (30-cm) diameter and is 4 feet (1.2 m) above the 12-inch (30-cm) inlet. The inlet vacuum is 6-inch (150-mm) mercury. The outlet pressure is 20 psig (140 kPa, gage). What power does the pump add to the fluid?

11. In a centrifugal pump test, the discharge gage reads 100 psig (690 kPa, gage), and the suction gage reads 5 psig (34 kPa, gage). Both read pressure above atmospheric. The gage centers are at the same level. The suction diameter is 3 inches (7.5 cm) and the discharge diameter is 2 inches (5 cm). Oil (SG = 0.85) flows at 100 gal/min (0.006 m^3/s). With no losses, what is the pump power?

Hydraulic Grade Line

12. Water is pumped at 2.00 ft^3/sec (0.06 m^3/s) through the system shown. Plot a graph of hydraulic grade versus position along the pipe.



[42]There are two definitions of the Weber number. The alternate definition is the square root of Eq. 17.180. In similarity problems, it does not make any difference which definition of the Weber number is used.