# PPI Guide to Significant Digits and Rounding Numbers

The NCEES rounds answer choices to the nearest significant digit so that an examinee will not know that he or she made an error simply because none of the answer choices match exactly what the examinee calculated. For instance, if the examinee calculated the answer to be 13.667 and the answer choices were 12.833, 14.425, 15.410, and 15.891, then the examinee would know immediately that his or her calculation was incorrect. On the other hand, if the answer choices were rounded to two significant digits—13, 14, 15, and 16—the examinee wouldn't be given any hints as to the accuracy of his or her calculation.

Usually two significant digits are sufficient to distinguish the choices, although sometimes three or more are necessary. For example, if the answer choices are 100.36, 100.44, 100.68, and 100.74, further rounding cannot be done because that would make the answer choices 100.4, 100.4, 100.7, and 100.7. The number of significant digits necessary in the answer choices should determined by the number of significant digits given in the problem statement. As a rule of thumb, the smallest number of significant digits in the problem statement determines the largest number of significant digits that can be in the answer choices.

In calculations, significant digits are important only when **reporting** a number. You should use all available digits, both significant and insignificant, during **intermediate calculations**, and round to the correct number of significant digits only when reporting the final result. If an intermediate calculation needs to be reported, it should be rounded **only for reporting purposes**. The rounded number should not be used in any further calculations.

**Rules of Significant Digits**

**1.** All digits 1 through 9 are significant.

If the mass of an object is measured as 12.3 g, then we know the mass is exactly between 12.2 g and 12.4 g. There are three significant digits in 12.3.

If the mass of an object is measured as 12.3456 g, then we know the mass is exactly between 12.3455 g and 12.3457 g. There are six significant digits in 12.3456.

**2.** A 0 is a significant digit when it lies between two nonzero digits.

The quantities 306, 3.06, 30.6, and 0.306 all contain three significant digits since the 0 between the 3 and the 6 is significant.

**3.** A terminal 0 to the right of a decimal point is significant.

The quantities 279.0, 27.90, and 2.790 all contain four significant digits. 0.2790 and 0.27900 have four and five significant digits, respectively.

**4.** A 0 used to fix a decimal point in a number less than 1 is not a significant digit.

The quantities 0.456, 0.0456, and 0.00456 all have three significant digits.

**5a.** A 0 used as a placeholder (following significant digits) is most often not a significant digit.

The quantity 19,000 has two significant digits, and the quantity 2030 has three.

**5b.** A 0 used as a placeholder (following significant digits) is significant in the following situation.

If three answer choices have 3 significant digits, and the fourth answer choice is a number containing a zero as a placeholder, the fourth answer choice is considered to have 3 significant digits. For example, (A) 192, (B) 198, (C) 204, (D) 210.

**5c.** In order to indicate placeholder 0's as significant digits, use scientific notation.

To give 19,000 three significant digits, write $1.90 \times 10^4$.

**6.** When multiplying and dividing, the number of significant digits in an answer should equal the least number of significant digits in any one of the numbers being used in the calculation.[1]

The product (0.203)(0.45) will be recorded with two significant digits.

**7.** When applying a trigonometric function, the number of significant digits in an answer should equal the least number of significant digits in any one of the numbers being used in the calculation.[2]

In evaluating sin $kx$, where $k = 0.097\ \text{m}^{-1}$ and $x = 4.73$ m, the answer should have two significant digits.

**8.** When applying logarithms, use the following rule. Retain in the mantissa (the number to the right of the decimal point in the logarithm) the same number of significant digits as there are in the number whose logarithm you are taking.

$\log 25 = 1.40$ and $\log 0.00581 = -2.236$

**9.** When adding or subtracting numbers, the result is rounded off to the decimal place of the least accurate component.

The sum 12 + 123.643 = 125.643 is rounded to 126.

**10.** The only time significant digits must be considered is when dealing with *measured* quantities. **Exact** or **defined** numbers should be considered to have an infinite number of significant digits. These are numbers that would not affect the accuracy of a calculation.

Consider the following examples.

- In the equation $d = 2\pi r$, 2 and $\pi$ have an infinite number of significant digits. Note that in a calculation, the significant digit to which $\pi$ is rounded must be kept in mind.
- In the conversion $2.54\ \frac{\text{cm}}{\text{in}}$, 2.54 also has an infinite number of significant digits since 1 inch is precisely defined now to equal 2.54 cm.[3]
- Interestingly, the speed of light is also now a defined quantity. By definition, the value is 299,792,458 meters per second.
- If we say that you have 100 apples, 100 has an infinite number of significant digits. (In general, whole number quantities have an infinite number of significant digits. On the other hand, if a distance is measured to be 100 mi, then 100 has only one significant digit as stated.)

**Examples: Identifying the Number of Significant Digits**

| example | number of significant digits | comment |
|---|---|---|
| 453 | 3 | All non-zero digits always significant. |
| 5057 | 4 | Zeros between non-zero digits are significant. |
| 5.00 | 3 | Zeros to the right of a decimal point are significant. |
| 0.007 | 1 | Zeros used to fix a decimal point are not significant. |
| 210 | 2 or 3 | “Placeholder” zeros are usually not significant. |

**Examples: Rounding to Two Significant Digits**

0.0141 = 0.014
196 = 200
10.8 = 11
19,451 = 19,000

**Rules of Rounding Numbers**

Throughout the course of your education, you may learn a number of different rules for rounding numbers. At PPI, we use the following rules in rounding numbers.

1. If the remaining digit to the right of the digit to be rounded is less than 5, drop the remainder. Rounding to one decimal place, the number 6.2467 becomes 6.2.
2. If the remaining digits to the right of the digit to be rounded are greater than 5, drop the remaining digits and increase the digit to be rounded by 1. (We are comparing each of the remaining digits with 5 followed by the appropriate number of zeros. Thus if the first remaining digit is equal to 5, we compare the next remaining digit to zero and so on.)

   Rounding to one decimal place, the number 6.2583 becomes 6.3.
3. If the remaining digits to the right of the digit to be rounded is exactly 5, drop the 5 and increase the digit to be rounded by 1.[4]

   Rounding to one decimal place, the number 6.25 becomes 6.3 and the number 6.35 becomes 6.4.

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[1] The rule presented here is the standard rule that one learns in school. There is an alternate rule involving multiplication and division that has been shown to be more accurate. For more information, see www.angelfire.com/oh/cmulliss.

[2] In actuality, there really are no standard rules for trigonometric functions, especially when considering functions such as the tangent function or the inverse trigonometric functions. When making calculations involving angle measurement and trigonometric functions, one should use error analysis.

[3] In 1959, the U.S. National Bureau of Standards redefined the foot to equal exactly 30.48 centimeters. This definition was also adopted in Britain by the Weights and Measures Act of 1963, so the foot of 30.48 centimeters is now called the **international foot**.

[4] Note that the National Institute of Standards and Technology (NIST), among others, uses the following rule when the remaining digit is exactly 5. *Round to the closest even number*. Thus, when rounding to one decimal place, the number 6.25 is rounded to 6.2 (rounding down) and the number 6.35 is rounded to 6.4 (rounding up). While this rule is commonplace for consistency in statistical data, PPI does not use this rule.